Infinitesimal Action

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Abstract

As defined by our theories of motion, an object's velocity v(t) at a time t is the limit of its average velocities over shorter and shorter time intervals that converge on t. Velocity might get attributed to an object at a single time t, but the value of v(t) depends on the object's locations before and after t. Many authors conclude that v(t) is not really an instantaneous quantity but a feature of a temporally extended process. To attribute a velocity to an object is to provide a coarse-grained description of its locations during a short time-interval. This "process theory" of velocity stands in a long philosophical tradition, going back to the pre-Socratics, that denies the existence of states of motion. Since motion involves the occupation of different places at different times, any description of motion must be relational and cannot characterize the world at a single instant, we are told. This paper promotes a different account, an "infinitesimal action" view that regards an object's velocity as a genuinely instantaneous quantity that constrains its locations at infinitesimally later and earlier times. The main motivation for the infinitesimal action view—and really the only reason why anybody needs to write a philosophy paper about velocity—is that the process theory is at variance with our best physics. While its kinematic definition suggests that velocity is a feature of a temporally extended process, its dynamic role in our best theories of motion is that of an instantaneous quantity. Suppose a meteor strikes the surface of Mars. The impact of the meteor depends on its velocity at the moment it hits the surface. Its velocity determines its kinetic energy, which must be dispersed as heat for the meteor to come to a halt. If velocity were merely a summary of an object's location over time, as the process theory contend, then it is not clear how the meteor manages to do this. It is not as if the meteor remembers where it was previously so as to hit Mars in just the right way. In Newtonian mechanics, forces determine velocities, by causing objects to accelerate. Relativistic theories also admit cases where velocities determine forces. One example is the relativistic mass of an object, which increases as its velocity increases. Since its mass determines the force that the object experiences in a gravitational field, an object's velocity determines what force it experiences in the presence of other masses. In electrodynamics, the force that a magnetic field exerts on a moving mass is proportional to the velocity of the charge. Its velocity thus determines how strongly a charge couples to a magnetic or a gravitational field.

The basic idea of the infinitesimal action view is to read the definition of velocity the other way around, as defining an instantaneous quantity that imposes a logically necessary constraint on the object's locations at infinitesimally earlier and later times. This is the temporal analogue of the received view about dispositions. To say that an object is fragile is to say that it breaks in the closest possible world in which it is struck. Nobody thinks that this shows that fragility is a feature of an extended region of logical space. To say that a vase is fragile is to say something about how it actually is. But it cannot be that the vase is fragile

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in the actual world if there is no other possible world in which it is struck and breaks. The actual fragility of an object imposes a necessary constraint on what happens in neighboring worlds. In a similar way, the infinitesimal action view claims that the present velocity of an object imposes a necessary constraint on where it is at neighboring times.
The main benefit of the infinitesimal action view is that it allows us to take our physical theories at face value, by regarding velocities as instantaneous features. But the view also raises several questions of its own. For example, it is not clear that the constraints imposed by velocities can remain infinitesimal. Don't we get diachronic necessitation by integrating velocities over time? There is also a range of questions about what to say about objects that move on non-differentiable trajectories, and thus lack velocities altogether. The aim of this paper is to show that these challenges can be met, and that the infinitesimal action view of velocity is far more appealing than one might have thought at first.

Keywords: velocity, action at a distance, theories of motion