Beyond Causality: Mathematical Explanations as Constraints in Science

Chen Zhibin*[1]

[1]Catholic University of Leuven - Katholieke Universiteit Leuven – Belgium

Abstract

Certain necessities in science, particularly those that govern the structure of the world independently of empirical contingencies, challenge conventional causal accounts of explanation. Why, for instance, must there always exist two antipodal equatorial points on Earth with identical temperature and atmospheric pressure at any given moment? The answer to this question often refers to the Borsuk-Ulam theorem from algebraic topology: any continuous mapping from an nnn-dimensional sphere to nnn-dimensional Euclidean space must identify at least one pair of antipodal points. In this case, mathematics seems to reveal a form of necessity embedded in the continuity of physical properties, transcending causal reasoning. To account for such distinctive mathematical explanations (DMEs) to scientific possibilities and necessities, Marc Lange (2013, 2016, 2018) introduces the concept of explanations by constraints. These explanations resist assimilation into causal frameworks and, instead, reflect how mathematical principles impose modal constraints on the world. Lange situates these explanations within the modal conception of explanation, a framework introduced by Wesley Salmon (1985) that departs from the ontic and epistemic conceptions of explanation.

However, the explanation-by-constraints framework has drawn significant criticism. On one side, the "chaos of constraint terminology" has been a persistent point of contention (Antonovics and van Tienderen 1991). On the other, some (e.g., Skow 2016) argue that these DMEs remain rooted in counterfactual or causal framework. This debate revolves around , according to causalists, whether such explanations derive their power "by appealing to what the world must be like as a matter of mathematical necessity" or "by appealing to various contingent causal facts." (Jha et al. 2024) These critiques demand a deeper examination of mathematical explanations in science, particularly their role in articulating possibilities and necessities.

First, this paper distinguishes which aspects of distinctively mathematical explanations (DMEs), as indentified by mathematical realists, qualify as constraints. For example, optimality explanations in biology are often seen as paradigmatic cases of Lange's explanation-by-constraints. However, I contend that examples like the hexagonal structure of honeycombs or the prime-numbered lifecycles of cicadas, while frequently cited in debates about the indispensability of mathematics, do not constitute constraint-based explanations. Conversely, cases like the Königsberg bridges and strawberry distribution exemplify how non-empirical reasoning can determine outcomes, effectively countering critiques rooted in causal facts. These cases underscore the conceptual differences and levels between law-based constraints

*Speaker

and mathematical explanations of constraints. Building on this foundation, I engage with Ross's (2023) proposal of causal constraints as a subtype of explanation-by-constraints. This approach resolves some ambiguities in earlier debates and addresses gaps in discussions of causal explanations-the causal explanations can also be a kind of modal explanations as constraints. To illustrate this, I draw on examples from biology to explore how causal and mathematical constraints interact and complement each other in explanatory practices.

Through the lens of mathematical explanations as constraints, this paper investigates the role of mathematics in explaining modal knowledge within scientific disciplines. By clarifying the distinctions and connections among mathematical, causal, and law-based constraints, I demonstrate the unique explanatory value of mathematics in articulating scientific possibilities and necessities. As an example, this paper shows how ongoing debates in scientific explanation contribute to a deeper appreciation of how necessity and possibility are framed in scientific inquiry.

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